Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Partial Differential Equations

Subject Code: 5SC02PDE1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 20/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Solve $(D^2 - D')z = 0$. **(02)**

b. Classify the region in which the equation $(D^2 - 3DD' + D'^2)z = 0$ is hyperbolic. **(01)**

c. Find particular integral of $r - 2s + t = \sin x$. **(01)**

d. Find $DD'z$ if x and y in $z = z(x, y)$ is replaced by $u = \log x$ and $v = \log y$ **(01)**

e. Define: Reducible factor **(01)**

f. Find order and degree of the differential equation $5s^2 - 4rq = xyp$. **(01)**

Q-2 **Attempt all questions** **(14)**

a. If $(\beta D' + \gamma)^2$ is a factor of $F(D, D')$ then prove that $e^{\frac{\gamma y}{\beta}}[\phi_1(\beta x) + y\phi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where ϕ_1, ϕ_2 are arbitrary function of single variable ξ. **(07)**

b. Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cdot \cos ny$. **(04)**

c. Find general solution of $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a. If $(\alpha D + \beta D' + \gamma)$ is a factor of $F(D, D')$ with $\alpha \neq 0$, then prove that $e^{\frac{-\gamma}{\alpha}x}\phi(\beta x - \alpha y)$ is a solution of $F(D, D')z = 0$, where ϕ is arbitrary function . **(06)**

b. Solve the partial differential equation $(D^2 + 4DD' + 4D'^2)z = \sqrt{x - 2y}$ by general method. **(05)**

c. Find a partial differential equation by eliminating f and g from **(03)**

$z = f(x - 2iy) + g(x + 2iy)$

Q-3 **Attempt all questions** **(14)**

a. Classify and reduce to canonical form and then solve the partial differential equation $4r - t = 0$. **(06)**

b. Solve: $(x^2D^2 - 2xyDD' - 3y^2D'^2 + xD - 3yD')z = x^2y\sin(logx^2)$ **(06)**

c. Find the characteristics of $(sin^2x)r + 2\cos x\, s - t = 0$. **(02)**

OR

Q-3 **Attempt all questions** **(14)**

a. Convert the given partial differential equation in to Canonical form **(06)**

$$\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} = y.$$

b. Solve:$(x^2D^2 + 2xyDD' + y^2D'^2)z = (x^2 + y^2)^{\frac{n}{2}}$. **(04)**

c. i) Define: complementary function and particular integral **(04)**

ii) Classify the partial differential equation

$xyr - (x^2 - y^2)s - xyt + py - qx - 2(x^2 - y^2) = 0.$

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Derive Green's Identity. **(02)**

b. The Poisson integral formula can be obtained from ________________. **(01)**

c. Wave equation is considered in the Dirichlet boundary value problem. Determine whether the statement is true or false. **(01)**

d. Write down Heat equation in Spherical Co-Ordinates system. **(01)**

e. $u = (x^2 - y^2)$ is a solution of two dimensional Laplace equation. Determine whether the statement is true or false. **(01)**

f. Using which method one can solve second order non linear partial differential equation? **(01)**

Q-5 **Attempt all questions** **(14)**

a. Derive Laplace equation in cylindrical co-ordinate. **(07)**

b. Solve partial differential equation $r + 4s + t + (rt - s^2) = 2$ by Monge's method. **(07)**

OR

Q-5 **Attempt all questions** **(14)**

a. Solve partial differential equation $r + 4s + 3t = xy$ by Monge's method. **(05)**

b. Solve $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ by the method of separation of variable and show that the solution can be put in the form of $J_n(mr)e^{\pm(mz+in\theta)}$, where m is constant and $J_n(mr)$ is a Bessel's function of order n. **(05)**

c. Using method of separation of variable solve: **(04)**

$$u_x + 2u_y = 0, u(0, y) = 4e^{-2y}$$

Q-6 **Attempt all questions** **(14)**

a. State and prove Harnack's theorem. **(07)**

b. Solve the following boundary value problem in the half plane $y > 0$, **(07)**

described by

PDE: $u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty, y > 0$

BCs: $u(x,0) = f(x), \quad -\infty < x < \infty$

OR

Q-6 **Attempt all Questions** **(14)**

a. Solve interior Dirichlet problem for a function $u = u(r,\theta)$ for a circle and show that solution is of the form $\sum_{n=0}^{\infty} r^n (A_n \cos n\theta + B_n \sin n\theta)$ with A_n and B_n are constants. **(08)**

b. Define equipotential surface and show that the family of surfaces $(x^2 + y^2)^2 - 2a(x^2 - y^2) + a^4 = c$ can form an equipotential surface and find the general form of the corresponding potential function. **(04)**

c. State Maximum principle. **(02)**